Practical Design of Minimum-Weight Aircraft Structures for Strength and Flutter Requirements

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Several methods for sizing the finite elements of an aircraft structural idealization to achieve minimum-weight design under combined strength and flutter-speed requirements are developed and evaluated. Two basic categories are considered: methods based on a combination of energy principles and optimality criteria; and procedures employing numerical-search techniques. Drawing upon the experience gained from studies of both of these basic methods, a resizing algorithm is developed that employs a uniform-flutter-velocity-derivative optimality criterion for flutter-critical elements and the fully-stressed-design criterion for strength-critical elements. The final result is a practical, automated approach for dealing with large-scale idealizations having both structural and mass-balance design variables.

Introduction

IN recent years, there has been an increasing emphasis in the aerospace community on minimizing the analysis time required for the detailed design of aircraft structures. This effort has largely focused on the refinement and integration of existing analysis tools.[1,2] In addition, the obvious benefits of automated redesign procedures as a means of further accelerating the design cycle were recognized several years ago, and studies in this area have resulted in an efficient approach to the computerized design of minimum-weight structures for strength requirements.[3] Since flutter prevention often plays a significant role in the practical design environment, it is apparent that a redesign procedure which minimizes structural weight in the presence of combined strength and flutter constraints would provide a further improvement in the efficiency of the structural design process. This paper gives primary attention to approaches that can result in practical procedures to accomplish the flutter design aspects of this combined task within the framework of previously developed strength design procedures.

For aircraft structures in which the design is governed exclusively by strength requirements, the fully-stressed-design (FSD) method appears today to offer industry the most economical and reliable approach to optimizing structures having several thousand design variables. Although it cannot be proven to yield absolute minimum-weight material distributions in redundant structures subjected to multiple loading conditions, its application as an "optimality criterion method"[4] has yielded practical and efficient airfame designs.[5] What the method lacks in the rigorous determination of precise minimum-weight designs is certainly offset by its ease of implementation and application.

For flutter-critical lifting surfaces where requirements for flutter prevention interact with those for strength, the state of the art in automated structural optimization, recently reviewed by Stroud,[6] is not nearly as far advanced. Early techniques for dealing with a specific flutter-speed requirement for a critical flight condition relied largely on time-consuming trial-and-error methods applied to "fix" a strength-only-based design. More recently, several investigators have suggested and employed various mathematical search procedures[7-9] or methods based on optimality criteria.[10] However, no approach has yet been advanced that embodies the desirable features of the FSD method in a complete and practical resizing algorithm to handle both strength and flutter-speed requirements, and that offers a high level of confidence in the optimality of the design it produces.

To ensure this confidence in such an automated approach, results obtained from many candidate procedures are compared herein for a sample wing structure of intermediate complexity. Essentially, the methods studied fall into two basic categories. The first includes optimality criteria methods based on energy concepts; the second emphasizes direct weight reduction by using numerical-search methods in conjunction with the expression for derivatives of flutter speed with respect to the design variables.[8] Based on the results of this study, an optimality-criterion method that uses a resizing formula based on flutter-velocity derivatives has been selected for further development in a combined flutter and strength optimization program called FASTOP.

Example Wing Structure

To provide a proper balance between the desire to achieve complete realism and the need to conduct many trial calculations in a short period of time, the moderately complex example wing shown in Fig. 1 was selected for study. Only the primary two-cell symmetric box structure is modeled with a finite-element representation. The cover skins are treated as constant-thickness quadrilateral membranes; beam and rib webs are modeled as shear panels; and bar elements are in-

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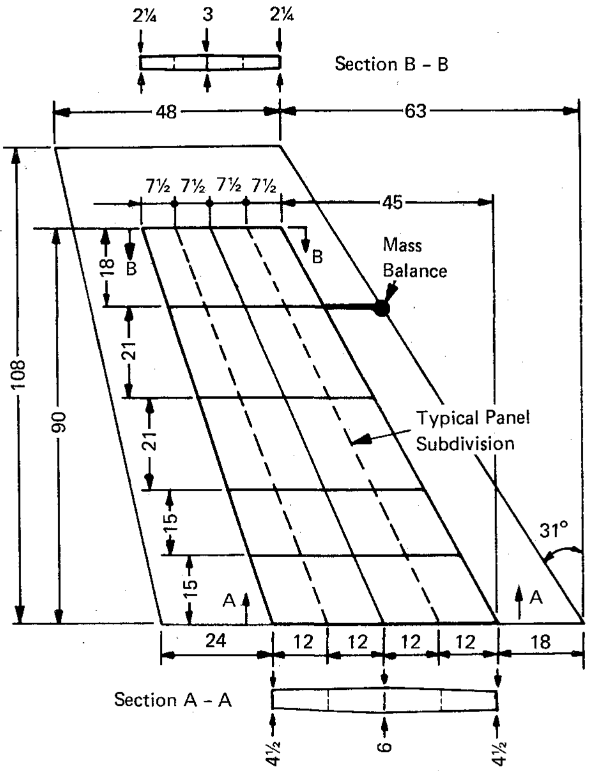

Fig. 1 Planform and primary structural arrangement for example wing. Note: all dimensions in inches except where otherwise noted.

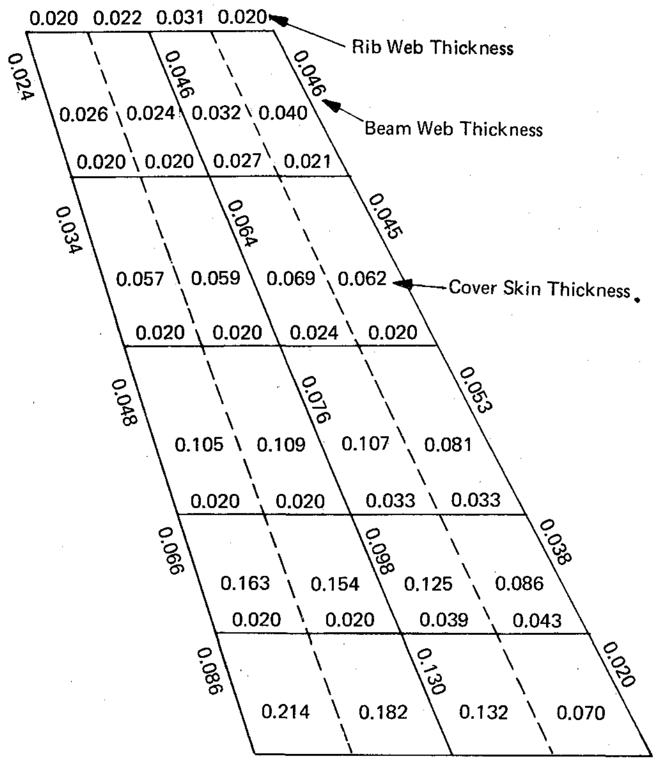

Fig. 2 Finite element gages for initial fully stressed example wing structure. Note: 1) all dimensions in inches; 2) minimum permitted gage = 0.020 in.; 3) bar elements connecting all upper and lower cover nodes are 0.10 in.[2]

troduced between upper- and lower-cover node points. The complete idealization contains a total of 100 structural design variables.

Initial sizing of the structures model was achieved by using the Automated Structural Optimization Program (ASOP)[11] with two representative flight loading conditions. The resulting "fully stressed design," shown in Fig. 2, is one in which each element is either critically stressed for one of the conditions (on the basis of average corner stresses for the membrane and shear panel elements), or is at a prescribed minimum manufacturing gage. Upper and lower cover thicknesses are the same since external loads were applied equally at opposite node points, and tensile and compressive allowable stresses were assumed equal.

The dynamics model was established by assigning equal parts of the weight of each finite element to the element's attaching node points. The node point weights were then scaled by factors of 2.0 for internal nodes and 3.0 for peripheral nodes to account approximately for the nonoptimum structural weight (e.g., fasteners, joints, etc.) and fixed-mass items (i.e., equipment and over-hanging structure) that would exist in a practical design. In determining the wing's vibration modes, the full 150 degrees of freedom of the structures model were employed.

Flutter speeds were computed using kernel-function unsteady aerodynamics for Mach zero at sea level, with generalized coordinates corresponding to the six lowest frequency modes of vibration. In the evaluation of all the methods presented, the goal was to resize this initial fully stressed design to achieve a 30% increase in flutter speed with a minimum increase in weight.

Energy-Based Optimality Criteria Methods

In the structures sense, an optimality criterion may be defined as a certain physical or mathematical property of a design for which it is suggested that the design is of near-minimum weight. In this first class of methods to be discussed, the optimality criteria, or procedures for satisfying such criteria, are based on structural energy concepts. The five methods presented differ primarily in the specific criteria they seek to satisfy. The simplest enforce a uniformly stressed state under 1) the inertial loading of the fundamental torsion mode of the structure; or 2) the combined aerodynamic and inertial loadings associated with selected instants in time during a flutter oscillation. More sophisticated criteria achieve 3) a state of uniform frequency derivatives in the torsion mode; 4) a state of uniform mean-strain-energy density in the flutter mode; and, finally, 5) a state of equal flutter-velocity derivatives. All of the criteria evolve from a general optimality criterion which it will be helpful to review first.

General Optimality Criterion

For a structure subjected to a single behavioral constraint function, F (e.g., a natural frequency, a deflection at a specific location, or a critical flutter speed), it can be mathematically proven that for a minimum weight design, the derivatives for this function with respect to the design variables, m_i (i.e., the weights of the finite elements comprising the design), must be equal. That is

$$\partial F/\partial m_i = \text{constant} \tag{1}$$

Such a proof is given by Berke,[4] who states the criterion as "...at minimum weight the change in behavior per change in weight, that is, the cost of improvement, is the same for every design variable."

This conclusion also follows from physical reasoning. Consider a design that precisely satisfies its only constraint and has two design variables with unequal behavioral derivatives. It would still be possible to satisfy the constraint precisely by increasing the weight of the element having the larger derivative while decreasing, by a larger amount, the weight of the element having the smaller derivative, thereby providing a net reduction in weight. It is evident, then, that the only design that could be one of minimum weight is one in which the derivatives are equal.

The general optimality criterion provides a standard of local optimality under the limited condition of a single design constraint. (The question of a local vs global optimum is not addressed in this work since it does not appear that practical automated procedures to deal with this problem are feasible at the present time.) However, realistic structural designs generally involve many constraints in combination (e.g., strength, flutter speed, minimum manufacturing gages) and thus the general optimality criterion for a single constraint, such as flutter speed, can only be applied to those design variables available to satisfy this constraint. In the multiple-constraint design problem, the separately governing optimality criteria must often be blended into a composite criterion which, to be of real value, must be achievable by a practical and efficient redesign procedure. These considerations have strongly influenced the development of the following energy-based methods as well as the numerical-search procedures discussed later. All of the energy methods are first summarized; their means of implementation and the results they produced are discussed subsequently.

Torsion Mode Fully Stressed Design

The torsion mode fully-stressed-design procedure is the simplest of those studied. It rests upon several assumptions that lead to a resizing scheme based on fully-stressed-design principles. The first is that a surface's flutter speed is approximately proportional to its torsional frequency. Next, it is assumed that for a given torsional frequency, a near-minimum-weight design is achieved when a state of uniform strain-energy density exists in the torsion mode. This assumption is drawn from an energy interpretation of the expression for the derivative of a structure's natural frequency with respect to one of its design variables, as follows: if the pertinent mode shape is normalized to make the generalized mass equal to unity, it can be shown (see, e.g., Ref. 12) that the frequency-derivative expression is

$$\frac{\partial\omega_n^2}{\partial m_i} = \{u\}^T\left(\frac{\partial[K]}{\partial m_i} - \omega_n^2\frac{\partial[M]}{\partial m_i}\right)\{u\} \tag{2}$$

where $[K]$ and $[M]$ are the structure's stiffness and mass matrices, respectively, $\{u\}$ is the column of mode shapes, ω_n is the corresponding natural frequency, and m_i is the ith design variable (i.e., finite element) weight. When the element's stiffness and mass matrices, $[K_i]$ and $[M_i]$, are proportional to its weight (an assumption made throughout this paper since this is usually the case in aircraft structural idealizations), the previous equations may be rewritten as

$$\frac{\partial\omega_n^2}{\partial m_i} = \frac{\{u_i\}^T[K_i]\{u_i\}}{m_i} - \frac{\omega_n^2\{u_i\}^T[M_i]\{u_i\}}{m_i} \tag{3}$$

where the subscript on the mode shape indicates that the only components included are those that deform or displace the ith finite element. This equation states that the rate of change of frequency squared with respect to a change in the element's weight is equal to the difference between its maximum double strain-energy density, SED_i, and its maximum double kinetic-energy density, KED_i. That is

$$\partial\omega_n^2/\partial m_i = \mathrm{SED}_i - \mathrm{KED}_i \tag{3a}$$

If it is now assumed that changes in element stiffness have a much greater influence in changing frequency than do changes in element mass (as is the case in structures having large amounts of fixed mass), the minimum-weight design associated with a state of uniform frequency derivatives is approximately equivalent to one having a state of uniform strain-energy density in the pertinent vibration mode.

A further consideration of the relationship between a state of uniform strain-energy density and a state of uniform stress leads to a very simple means of implementing an automated redesign procedure for approximately achieving the former state. Consider, for example, a finite element representing a rectangular shear panel having a working stress τ, a shear modulus G, and a weight density ρ. The element's double strain-energy density (by weight) is then $\tau^2/G\rho$. If a structure is comprised of a family of such elements, all of the same material and having the same allowable stress, it will thus exhibit a state of uniform strain-energy density when it is fully stressed for a particular loading condition. Together with the previous assumptions, a loose extension of this notion to a general wing-type structural arrangement suggests a procedure for optimizing a design for both strength and flutter by utilizing existing FSD methods. This involves the formulation of an additional loading condition corresponding to the inertial loads for the fundamental torsion mode of the surface, and the introduction of this loading condition in the redesign process along with the actual applied loading conditions. The scaling of this additional condition depends upon the desired increase in flutter speed for a redesign step. As the loads are increased, the number of elements that are resized to accommodate the flutter-speed requirement will also increase. The elements affected most will be those having the highest stresses or, approximately, the largest frequency derivatives.

Flutter Mode Fully Stressed Design

The flutter mode fully-stressed-design approach is an extension of the previous method to the complex flutter mode itself. It is based on the intuitive premise that what is desired is to make the design fully stressed (or of near-uniform strain-energy density) for a flutter oscillation cycle. Contrary to the situation existing in a normal mode where the loads acting on the structure are purely inertial and proportional to amplitude the loads associated with motion in the flutter mode are both inertial and aerodynamic, and depend upon phase as well as amplitude. Consequently, it is felt that a single pseudo-load condition in an FSD algorithm will not suffice for the flutter mode, and a family of loads, with each member corresponding to a different time in the oscillation cycle, should be defined. One such family is selected by considering the variation of the total strain energy of the structure during a cycle.

If $\{U\}$ is the complex flutter vector defining the displacements and relative phasing of the coordinates at some arbitrary initial time, and ω_f is the flutter frequency, it can be shown (see Appendix) that the strain energy, E_s, in the total structure is given by

$$E_s = \tfrac{1}{4}(a+b) + \tfrac{1}{4}(a^2+b^2+c^2-2ab)^{1/2}\sin(2\omega_f t - \varphi) \tag{4}$$

where

$$a = \{U_R\}^T[K]\{U_R\} \tag{5a}$$

$$b = \{U_I\}^T[K]\{U_I\} \tag{5b}$$

$$c = -2\{U_R\}^T[K]\{U_I\} \tag{5c}$$

and

$$\varphi = \tan^{-1}[(b-a)/c] \tag{6}$$

in which subscripts R and I indicate the real and imaginary parts, respectively, of the flutter vector. Thus, the total strain energy oscillates at twice the flutter frequency about a mean level given by $(a+b)/4$, and contrary to the case of vibration in a normal mode, never reaches a zero level. The pseudo-loading conditions selected in this study are those associated with the times of maximum, minimum, and mean strain energy that occur over one complete flutter oscillation cycle.

For the general case of an asymmetric wing cross-section, this requires eight conditions. However, for the sample wing structure discussed previously with its symmetric upper and lower covers, this reduces to four independent conditions that must be used in conjunction with the actual applied loading conditions in the FSD resizing method. Similar to the previous method, the scaling of the flutter mode loading conditions is based on the desired increase in flutter speed, but in this case, the entire family of conditions must be scaled by a common factor.

Uniform Frequency Derivatives in the Torsion Mode

The third method, while still relying on approximate proportionality between torsional frequency and flutter speed, treats the frequency optimization problem in a more exact manner than the first. The approach deals directly with both the strain and kinetic-energy-density distributions in the torsion mode, leading to a resizing formula for achieving a state of uniform frequency derivatives. Before proceeding with the development of the resizing equation, a basic concept will be developed for use here and in subsequent discussion.

Consider a structure under two general loading conditions that produce displacement shapes $\{\Delta^{(P)}\}$ and $\{\Delta^{(Q)}\}$. For the ith finite element, a "generalized" strain-energy density, SED_i, may be defined as

$$SED_i = \frac{\{\Delta_i^{(P)}\}^T [K_i] \{\Delta_i^{(Q)}\}}{m_i} \tag{7}$$

where the subscript on the displacement vector indicates that the only components included are those associated with the ith element. Now assume that the element is resized from a weight m_i to $\tilde{m}_i$ by a factor α, i.e.

$$\tilde{m}_i = \alpha m_i \tag{8}$$

If the internal loads remain unchanged after resizing (the same assumption that underlies the fully-stressed-design approach to structural optimization for strength requirements), then, for elements in which stiffness is proportional to the design variable

$$[K_i]\{\Delta_i^{(Q)}\} = [\tilde{K}_i]\{\tilde{\Delta}_i^{(Q)}\} \tag{9a}$$

and

$$\{\Delta_i^{(P)}\}^T [K_i] = \{\tilde{\Delta}_i^{(P)}\}^T [\tilde{K}_i] = \alpha \{\tilde{\Delta}_i^{(P)}\}^T [K_i] \tag{9b}$$

where a tilde (~) over a character indicates its value after redesign. Using these relationships, the generalized strain-energy density after redesign may be related to its original value by

$$\widetilde{SED}_i = \frac{\{\tilde{\Delta}_i^{(P)}\}^T [\tilde{K}_i] \{\tilde{\Delta}_i^{(Q)}\}}{\tilde{m}_i} = \frac{\{\tilde{\Delta}_i^{(P)}\}^T [K_i] \{\Delta_i^{(Q)}\}}{\alpha m_i} = \frac{1}{\alpha^2} SED_i \tag{10}$$

or, using a slightly different notation,

$$\frac{(SED_i)_{new}}{(SED_i)_{old}} = \left(\frac{m_{i_{old}}}{m_{i_{new}}}\right)^2 \tag{11}$$

That is, the generalized strain-energy density varies inversely with the square of the element's weight and if resizing is to be based on this energy quantity

$$m_{i_{new}} = m_{i_{old}} \left(\frac{(SED_i)_{old}}{(SED_i)_{new}}\right)^{1/2} \tag{12}$$

This relationship becomes directly applicable to the frequency optimization problem if the pertinent (torsional) mode shape $\{u_i\}$ is used in place of $\{\Delta_i^{(P)}\}$ and $\{\Delta_i^{(Q)}\}$ in Eq. (7).

To achieve equal derivatives, according to Eq. (3) or (3a), it is desirable to resize each element so that its new strain-energy density is

$$(SED_i)_{new} = (\partial\omega_n^2/\partial m_i)_{target} + (KED_i)_{new} \tag{13}$$

where $(\partial\omega_n^2/\partial m_i)_{target}$ is the desired uniform value of the derivative after redesign. The following rationale allows replacing the unknown quantity $(KED_i)_{new}$ by its value before redesign.

For elements in which the stiffness and mass matrices are linear in the design variable, the energy densities at constant frequency vary only because of changes in mode shape. However, the changes in mode shape that most significantly contribute to changes in an element's strain-energy density differ from those that change its kinetic-energy density. The former is affected exclusively by the relative motions of the boundary nodes of the element; the latter depends only on the absolute motions. While the relative motions change significantly with element gage changes (in inverse proportion, if the internal loads remain unchanged), the absolute motions change to a much lesser degree. Thus, it appears reasonable to assume that, for small design changes, the mode shapes of the kinetic-energy-producing terms remain unchanged. Using this assumption and combining Eqs. (12) and (13), the following resizing formula is obtained

$$m_{i_{new}} = m_{i_{old}} \left(\frac{(SED_i)_{old}}{(\partial\omega_n^2/\partial m_i)_{target} + (KED_i)_{old}}\right)^{1/2} \tag{14}$$

The value selected for the target derivative depends on the desired increase in frequency (or flutter speed) for a redesign step.

Uniform Mean-Strain-Energy Density in the Flutter Mode

An attempt to approximate an expression for the flutter-velocity derivatives of a structure in a form that lends itself to a simple energy interpretation leads to another optimality criterion, one that does not depend on assumed proportionality between torsional frequency and critical flutter speed. Interestingly, the criterion that follows from this development is the same as that adopted by Siegel.[10]

The development closely parallels the approach used by Rudisill and Bhatia[8] in their formulation of the precise flutter-velocity derivatives. However, by assuming that design changes have negligible effect on the flutter mode shape (i.e., $\partial\{U\}/\partial m_i = 0$) and by introducing the transposed complex conjugate, $\{\bar{U}\}^T$, of the flutter vector instead of $\{V\}^T$ (the associated row vector of the eigenvector $\{U\}$), the following simplified, but inexact, expression may be obtained

$$\frac{\partial V_f}{\partial m_i} \approx \frac{V_f}{2\omega_f^2} \{\bar{U}_i\}^T \left(\frac{[K_i]}{m_i} - \omega_f^2 \frac{[M_i]}{m_i}\right) \{U_i\} \tag{15}$$

where V_f is the flutter speed. In deriving this equation, the flutter vector has been normalized such that

$$\{\bar{U}\}^T([M] + [A])\{U\} = 1 \tag{16}$$

where $[A]$ is the complex aerodynamic force matrix.

If the kinetic-energy terms are neglected, as in the first two methods, Eq. (15) simplifies further to

$$\frac{\partial V_f}{\partial m_i} \approx \frac{V_f}{2\omega_f^2} \frac{\{\bar{U}_i\}^T [K_i] \{U_i\}}{m_i} \tag{17}$$

which suggests a strain-energy-type optimality criterion. By observing the equality

$$\frac{\{\bar{U}_i\}^T[K_i]\{U_i\}}{m_i} = \frac{\{U_{i_R}\}^T[K_i]\{U_{i_R}\}}{m_i} + \frac{\{U_{i_I}\}^T[K_i]\{U_{i_I}\}}{m_i} \tag{18}$$

in relation to Eqs. (4) and (5), the achievement of a state of uniform flutter-velocity derivatives, as approximated by Eq. (17), is equivalent to making the mean-strain-energy density of all elements equal. Resizing to satisfy this criterion is accomplished by again assuming that an element's strain-energy density varies inversely with the square of its weight, so that

$$m_{i_{\text{new}}} = m_{i_{\text{old}}} \left(\frac{(\text{SED}_i)_{\text{old}}}{(\text{SED}_i)_{\text{target}}} \right)^{1/2} \tag{19}$$

where in this case the SED_i terms correspond to the sum of the two energy quantities appearing in Eq. (18). As before, the target value of this energy quantity must be adjusted to satisfy the desired increase in flutter speed.

Uniform Flutter-Velocity Derivatives

All of the methods discussed thus far involve certain assumptions which, while sometimes difficult to justify, lead to optimality criteria that can be satisfied by employing familiar concepts, the hope being that even when an approximate criterion is satisfied, the design will be close to one of minimum weight. None of the methods, however, takes advantage of the knowledge of the precise flutter-velocity derivatives and the ultimate desire to make these quantities uniform. In this last energy-based method, it is shown that the exact derivative expression can be cast into a new form that is identifiable in terms of generalized energy quantities of the type discussed previously. By then extending some of the concepts used earlier, it is possible to obtain a logical resizing formula for achieving a state of uniform flutter-velocity derivatives among the structural elements. The development proceeds as follows:

With the flutter-mode vector $\{U\}$ and its associated row vector $\{V\}^T$ normalized such that

$$\{V\}^T([M]+[A])\{U\}=I \tag{20}$$

Rudisill and Bhatia's final flutter-velocity derivatives[8] may be expressed in the following form

$$\frac{\partial V_f}{\partial m_i} = -\frac{V_f}{\omega_f^2}\Bigg[\tfrac{1}{2}Re(\{V\}^T[\partial[K]/\partial m_i - \omega_f^2\partial[M]/\partial m_i]\{U\}) - \frac{Im(\{V\}^T[\partial[K]/\partial m_i - \omega_f^2\partial[M]/\partial m_i]\{U\})}{Im(\omega_f^2\{V\}^T(\partial[A]/\partial k)\{U\})} \times \left(\frac{\omega_f^2}{2} Re(\{V\}^T(\partial[A]/\partial k)\{U\}) + \frac{\omega_f^2}{k} \right) \Bigg] \tag{21}$$

where k is the reduced frequency, $b\omega_f/V_f$, in which b is the reference wing semichord. If the ith element's stiffness and mass matrices vary linearly with its design variable, a new version of Eq. (21), similar in form to the previous energy expressions, can be obtained:

$$\begin{aligned}\frac{\partial V_f}{\partial m_i} = &\left(\frac{V_f}{2\omega_f^2} \Bigg[\frac{\{V_R\}^T[K_i]\{U_R\}}{m_i} - \frac{\{V_I\}^T[K_i]\{U_I\}}{m_i} - C\left(\frac{\{V_R\}^T[K_i]\{U_I\}}{m_i} + \frac{\{V_I\}^T[K_i]\{U_R\}}{m_i} \right) \Bigg] \right) \\ &- \left(\frac{V_f}{2} \Bigg[\frac{\{V_R\}^T[M_i]\{U_R\}}{m_i} - \frac{\{V_I\}^T[M_i]\{U_I\}}{m_i} - C\left(\frac{\{V_R\}^T[M_i]\{U_I\}}{m_i} + \frac{\{V_I\}^T[M_i]\{U_R\}}{m_i} \right) \Bigg] \right)\end{aligned} \tag{22}$$

or

$$\frac{\partial V_f}{\partial m_i} = \text{SED}_i^* - \text{KED}_i^* \tag{22a}$$

where the subscripts R and I indicate the real and imaginary components, respectively, of the flutter vector and its associated row vector, and where C is a real quantity defined by

$$C = \frac{Re(\{V\}^T(\partial[A]/\partial k)\{U\}) + 2/k}{Im(\{V\}^T(\partial[A]/\partial k)\{U\})} \tag{23}$$

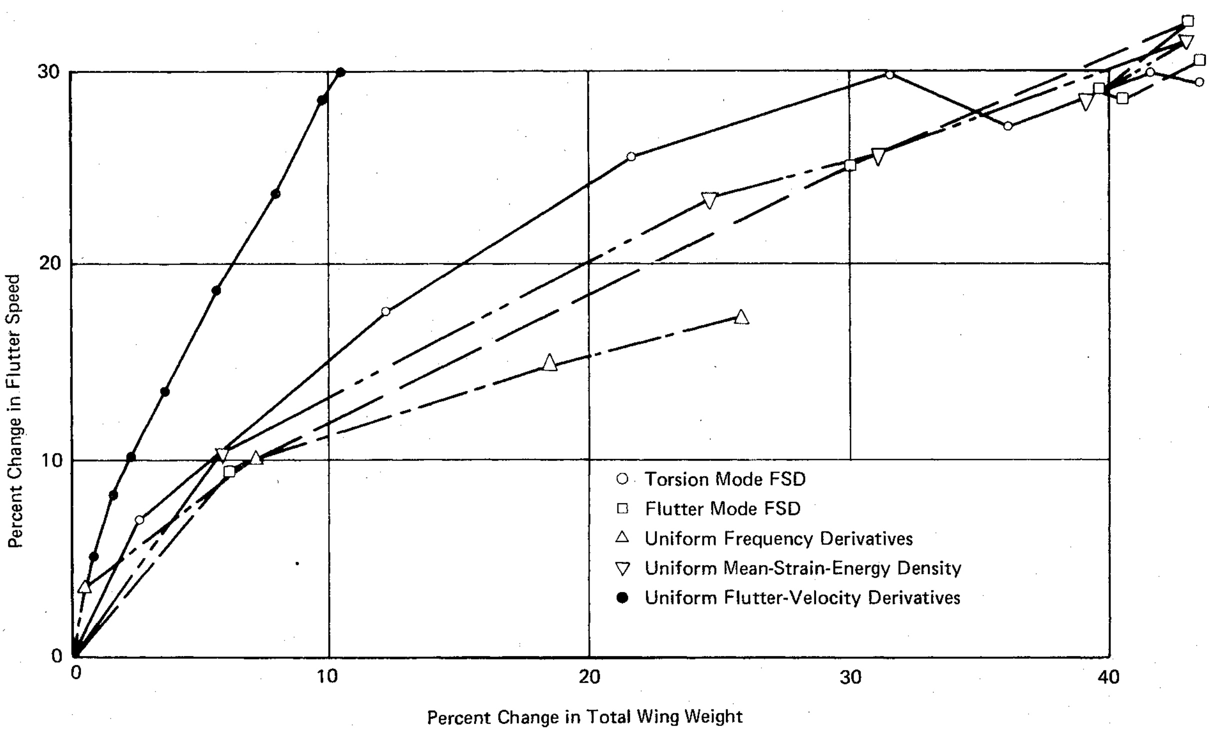

Fig. 3 Results of five energy-based methods.

In Eq. (22), terms have been grouped into two categories. The first, SED_i^*, includes what may be interpreted as a linear combination of generalized strain-energy-density terms. The second category, KED_i^*, contains a similar set of generalized kinetic-energy-density terms. It should be noted that some liberty has been taken to simplify notation with regard to the pre and post-multiplying displacement shapes. The subscript i is implied and the components that are included in the specific rows and columns are only those that multiply $[K_i]$ or $[M_i]$.

By pursuing the same line of reasoning that was applied in the development of a resizing formula for achieving uniform frequency derivatives, Eq. (14) may be translated into the following form for dealing directly with the flutter-optimization problem

$$m_{i_{\text{new}}} = m_{i_{\text{old}}} \left(\frac{(SED_i^*)_{\text{old}}}{(\partial V_f / \partial m_i)_{\text{target}} + (KED_i^*)_{\text{old}}} \right)^{1/2} \tag{24}$$

where $(\partial V_f / \partial m_i)_{\text{target}}$ is the desired uniform value of the flutter-velocity derivatives. In writing this expression, it is assumed that the combination of generalized strain-energy-density terms, SED_i^*, varies inversely with the square of the ith element's design variable. This follows from the premise that internal loads corresponding to any of the displacement shapes defined by $\{U_R\}$, $\{U_I\}$, $\{V_R\}$, or $\{V_I\}$ remain unchanged during redesign and from the approximation that the coefficient C remains constant.

Evaluation of Energy-Based Methods

A summary of results obtained by applying each of the preceding methods to the sample wing structure is presented in Fig. 3. The weight and flutter-speed increases corresponding to a design point are expressed as percentages of the total weight and flutter speed of the initial strength-governed design. Nonoptimum factors were not applied to the incremental weights associated with resized elements.

Both the torsion mode and flutter mode fully-stressed-design procedures were implemented with ASOP by introducing additional applied loading conditions in conjunction with the actual applied loads. The proper scaling of these supplementary conditions became a matter of judgment and experience with no rigorous scaling algorithm. It was desired to reach the specified 30% increase in flutter speed in small increments. For each step, the applied loading condition(s) associated with the appropriate mode shape of the previous design points were held constant, and the structure was resized by the fully-stressed-design algorithm to achieve the desired stress state. Each resulting design thus reflected the interaction between strength and flutter requirements. When the desired flutter speed had been achieved, the procedure was continued until design changes became acceptably small. Figure 3 shows that both methods tended toward convergence at designs requiring weight increases of about 43% for the desired increase of 30% in flutter speed. A rather dramatic indication of the weakness of both of the methods was the existence of an intermediate design that satisfied all constraint requirements with less than a 32% weight increase. This clearly demonstrated the incorrectness of the associated optimality criteria of these methods.

Resizing in accordance with the uniform-frequency-derivatives approach was accomplished by combining ASOP with a special routine that computed the strain and kinetic-energy densities of the elements for the torsion mode. For each element, its new size was taken as the larger of the values obtained by the resizing formula of Eq. (14) or the stress ratio $(\sigma_i / \sigma_{i\,\text{allowable}})$ for a fully stressed design. As with the previous methods, it was desired to increase the flutter speed in small increments, and trial-and-error procedures were required to select the proper values for the target frequency derivatives. To achieve each new design point, several iterations were performed to account for interaction between strength and frequency requirements. For each of these iterations, adjusted displacement shapes (for use in the calculation of the SED_i terms) were computed under the assumption that external inertial loads associated with the torsion mode remained unchanged. The target derivative and the KED_i terms were, however, assumed constant from one iteration to the next. After each new design was achieved, a new torsion mode and flutter speed were computed and the target derivative and the KED_i terms were revised.

As shown in Fig. 3, except for a very small weight increase initially, this method gave substantially greater weight increases than the previous methods. Based on this poor trend, evaluation of the procedure was aborted before a fully converged design yielding the desired 30% flutter speed increase was attained. However, as a matter of interest, the results were examined to determine the method's value in dealing with a frequency constraint, that is, the actual constraint for which the procedure was derived. It was observed that at each new design point the frequency derivatives of the elements resized by Eq. (14) displayed a high level of uniformity, even though the resizing was based on the mode shape of the previous design. These results and the basic simplicity of the resizing equation indicate the method's potential for dealing with structural optimization problems involving such a constraint.

To expedite the implementation of the final two energy methods, two simplifications were made to arrive at the desired flutter speed. First, interaction between flutter and strength requirements was accounted for by simply not allowing an element's size to fall below its value at the initial fully stressed design. Second, only a single redesign took place for each new design point; thus, in contrast with the previous methods, each new design did not necessarily satisfy its optimality criterion based on modal characteristics of the previous design. Only after the desired 30% flutter speed increase was obtained, was an attempt made to achieve a converged design.

The evaluation of the procedure based on the uniform mean-strain-energy density optimality criterion led to a final design in the same neighborhood as the two FSD-type methods. Each intermediate redesign point was obtained through the use of Eq. (19), with the target value of SED_i obtained by trial and error.

The last method, which seeks uniformity in the precise flutter-velocity derivatives by use of an energy-based resizing algorithm, required a much smaller weight increase than all previous methods. Figure 3 shows that the final, approximately converged design achieved the desired 30% flutter-velocity increase with a weight increase of only 10.5%. Resizing was accomplished through the use of Eq. (24) where the target value of the flutter-velocity derivative for each redesign was chosen to effect small increases in flutter speed. In any redesign step, those elements for which the term under the radical of this equation were negative were held at their previous gages. A very significant aspect of the results was the fact that each new design corresponding to a point on the appropriate curve in the figure exhibited a very high degree of uniformity among the flutter-velocity derivatives of the flutter-critical elements. (This was not the case with any of the preceding methods.) Also, since each design was obtained with only a single resizing step, it appears that the resizing formula embodies excellent convergence characteristics.

The results of this investigation show that an optimality criterion based on the exact flutter-velocity-derivative expression can lead to a final design that is substantially lighter than those based on simplified criteria. While a resizing formula was developed for satisfying this criterion, it relies on strain-energy-related quantities and is limited to the redesign problem where only structural design variables are considered. Therefore, to consider mass-balance variables, an additional or more general redesign formula is needed. The numerical-search methods discussed next, as well as the final method selected for development, have the capability for dealing with mass balance.

Numerical-Search Procedures

Paralleling the evaluation of the energy-based methods was the development and study of several numerical search procedures that all employ the previously referenced expression for the precise flutter-velocity derivatives. A major distinction between these procedures and those already discussed is that the numerical-search methods do not rely on the definition and enforcement of an optimality criterion. Instead, concepts of travel in design space are employed to seek out a near-minimum-weight design that satisfies the flutter-speed constraint without compromising strength requirements. Before discussing the particular methods that were examined, it is desirable to present some basic concepts regarding travel in design space as they apply to the flutter/strength optimization problem.

Design Travel in Weight Space

For purposes of this discussion, a weight space is defined in terms of the design variables (element weights, m_i) that are active in the optimization process. To enable visualization, a simple two-design-variable problem with typical lines of constant flutter velocity and constant weight is illustrated in Fig. 4. With $\bar{V}_f$ and $\bar{m}$ defined as the flutter-velocity-gradient and mass-gradient vectors, respectively, at design point A, it may be observed that the vector $\bar{p}=\bar{V}_f-\gamma\bar{m}$ will (for the general higher-order design space) lie in a plane perpendicular to both the mass hyperplane and flutter-velocity hypersurface at that point, its direction being dependent on the value of the scalar quantity γ. A redesign step in the direction of $\bar{p}$ may be represented by a vector $\overline{\delta m}=K(\bar{V}_f-\gamma\bar{m})$ where K controls the step size. In the general multi-variable case, $\overline{\delta m}$ represents a redesign vector whose elements are the change in weight, δm_i, of each design variable. Realizing that all components of the weight-surface gradient are unity, the change in weight of the ith design variable is

$$\delta m_i = K[\partial V_f/\partial m_i - \gamma] \tag{25}$$

In the vicinity of a design point, the flutter-velocity derivatives may be used to estimate change in flutter speed as a linear function of these incremental weights; that is

$$\delta V_f = \sum_i (\partial V_f/\partial m_i)\delta m_i \tag{26}$$

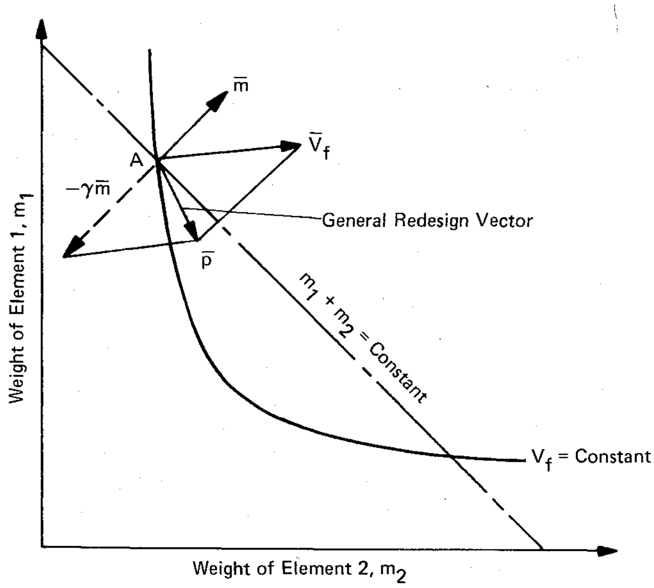

Fig. 4 Two-design-variable space.

For the special case of $\gamma=0$ in Eq. (25), it is evident that the redesign vector will be parallel to the flutter-velocity gradient vector and the incremental weight changes will be

$$(\delta m_i)_G = K(\partial V_f/\partial m_i) \tag{27}$$

where the subscript G denotes a gradient mode of travel. On the other hand, if the redesign vector is tangent to the flutter-velocity hypersurface, then

$$(\delta V_f)_T = \sum_i \left(\frac{\partial V_f}{\partial m_i}\right)(\delta m_i)_T = 0 \tag{28}$$

in which T is introduced to indicate a tangential mode of travel. Substituting the δm_i of Eq. (25) into Eq. (28) leads to

$$\sum_i \partial V_f/\partial m_i [\partial V_f/\partial m_i - \gamma_T] = 0 \tag{29}$$

from which the direction scalar, γ_T, is obtained as

$$\gamma_T = \frac{\sum_i \left(\frac{\partial V_f}{\partial m_i}\right)^2}{n\left(\frac{\partial V_f}{\partial m_i}\right)_{\text{average}}} \tag{30}$$

where n is the total number of active design variables.

As noted previously, the redesign vector $\overline{\delta m}$, lies in a plane that is perpendicular to the mass hyperplane. Thus, travel in the particular tangent direction defined by the redesign equation

$$(\delta m_i)_T = K[\partial V_f/\partial m_i - \gamma_T] \tag{31}$$

will assure the maximum rate of weight change that can be achieved while maintaining a constant flutter speed. These gradient and tangent modes of travel form the basis for the numerical-search procedures employed for flutter redesign.

The requirement that a structure remain strength-adequate during flutter redesign is represented in weight space by the existence of strength-constraint hypersurfaces that limit the permitted direction or amount of design travel. These surfaces define the condition in which the stress ratio of a design element is unity (i.e., it is fully stressed). Additional planar constraint surfaces represent minimum-gage requirements dictated by manufacturing considerations. To avoid violating these constraints, the weight changes indicated by Eq. (25) must be truncated so that no element size is permitted to be smaller than that required to meet its strength or minimum-gage requirements. In the present evaluation of methods, these constraints were held constant at the element sizes of the initial fully stressed design, and were not updated to account for load redistribution following redesign for flutter.

Of the four methods summarized in the following subsections, the first two fall in the category of methods for initially achieving the prescribed flutter speed. The latter two are methods for minimizing structural weight after the flutter speed target is achieved. A summary of the results obtained by applying numerical-search techniques to the sample wing structure is presented in Fig. 5. For comparison purposes, this figure also includes the previous results for the optimality-criterion method that uses the resizing formula based on exact flutter-velocity derivatives.

Flutter Velocity Gradient Redesign

Gradient redesign in accordance with Eq. (27) presented a simple procedure for initially achieving the specified 30% increase in flutter speed, although results indicate it was relatively inefficient in comparison with the uniform-flutter-velocity-derivative approach. The disadvantage of the

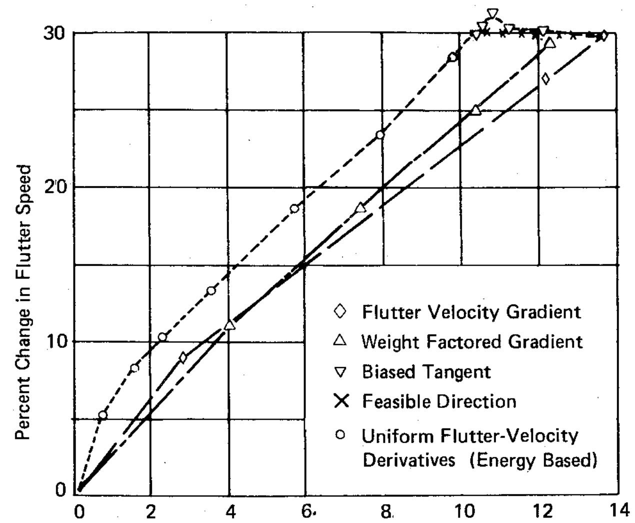

Fig. 5 Results of numerical-search methods.

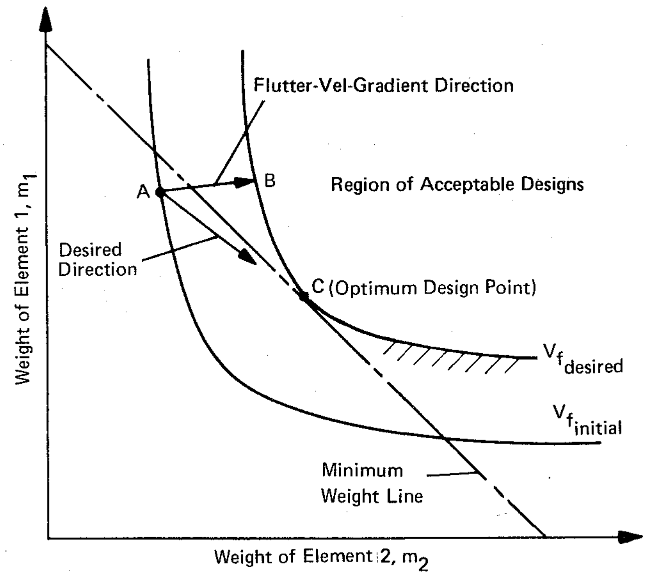

Fig. 6 Two-design-variable space showing deficiency of gradient travel.

gradient redesign procedure is apparent in the two-design-variable problem of Fig. 6. In this illustration, the desired minimum weight design is at C, the point of tangency between the required constant-velocity curve and a constant-mass line, and consequently, the point where the flutter-velocity derivatives are equal. However, the gradient direction at initial design point A will lead to design point B remote from the optimum.

Upon examining the flutter-velocity derivatives of the final design achieved by this approach, it was observed that several of the lighter elements had derivatives well below the mean for all elements that were resized even though these lighter elements had large derivatives at the starting design. This indicated that too much weight had been added to these members. The tendency of gradient resizing to overemphasize light elements is explained by examining the resizing expression, Eq. (27), which indicates that incremental weight is only a function of an element's derivative value. However, it was consistently noted in the course of this evaluation that the ability of an element to sustain its effectiveness throughout redesign is a function of its weight, a parameter that is not included in the gradient resizing equation.

Weight Factored Gradient Redesign

In an attempt to compensate for the deficiency of the previous approach, a revised algorithm was evaluated in which the incremental weight added to each element was factored by the element's total weight:

$$\delta m_i = K m_i (\partial V_f / \partial m_i) \qquad (32)$$

This approach to some extent alleviated the oversizing of light elements and showed improvement over the preceding method in the total weight increment for the final design, Fig. 5.

Biased-Tangent Method

The biased-tangent method was used to minimize the weight of the example wing structure after the required flutter speed had been initially achieved using the gradient method. It was desired to maintain constant flutter speed during the process. The method for traveling tangent to the velocity hypersurface defined by Eq. (31) was employed initially, but it was found that the flutter speed tended to drift during the redesign process, as illustrated in Fig. 7. This drift was reduced by decreasing the step-size parameter, K, but this imposed a severe penalty on computation time. To minimize the drifting phenomenon while maintaining an efficient step size, a modified redesign direction was employed to compensate for the apparent curvature of the flutter-velocity hypersurface by calling for a slight change in flutter speed even though none would be needed according to a linear approximation. The net effect of this compensation was to bias the redesign vector with respect to the tangent direction, as indicated by the angle α in the figure.

The specific approach used was as follows: if, after the initial resizing step in the tangent mode, a reduction in flutter speed, $-\delta V_f$, was observed, than a compensation value of $2\delta V_f$ was specified for the next redesign. The factor of two was applied to correct both the initial drift and anticipated drift of the next redesign. Using the linearized approximation of Eq. (26) in conjunction with Eq. (25), it was possible, using an iterative procedure to account for the presence of strength constraints, to obtain a value of γ that satisfied both equations with the step-size factor, K, held fixed. In subsequent redesign steps, the value of δV_f used to bias the tangent was determined by a more general procedure in which the anticipated drift was based on the behavior of previous redesign steps; specifically

$$\delta V_{f_{j+1}} = (V_{f_{\text{target}}} - V_{f_j}) + [\delta V_{f_j} - (V_{f_j} - V_{f_{j-1}})]$$

As shown in Fig. 5, implementing the biased-tangent method resulted in a design having essentially the same weight as that achieved by the uniform-flutter-velocity-derivative approach. However, difficulty was experienced in choosing values for the step-size parameter, K, to achieve satisfactory convergence. Moreover, it was noted that the flutter-velocity derivatives of light structural elements tended to oscillate in the final redesign cycles, resulting in a lack of convergence in the design of those particular elements. This latter tendency was reduced by truncating the redesign of all elements in the final cycles so that the maximum design change did not exceed a specified percentage.

Feasible-Direction Method

The feasible-direction method, which has been explored in some previous flutter-optimization studies,[9,13] is based on the work of Zoutendijk.[14] Though the fundamental objectives of this method and the preceding one are similar, the procedures for determining the direction of the redesign vector are different. In its specific application to the problem of weight minimization at constant flutter speed, the feasible-direction method constrains design travel within a sector whose boundaries are the mass hyperplane and tangent to the velocity hypersurface at a design point. The direction of the redesign vector is determined by solving a linear suboptimization problem by the simplex method. The procedure requires the user to specify step size and a "pushoff" factor which is analogous to the bias angle, α, of the preceding method. As indicated in Fig. 5, the final design obtained by applying this approach to the sample problem, following velocity-gradient redesign, was very close to the design obtained by the biased-tangent method. However, although both procedures yielded good results, considerable difficulty was experienced in developing an efficient automated step-size procedure. It is apparent that this problem is associated with the fact that both methods attempt to accomplish design travel close to a curved velocity hypersurface with knowledge of only the tangent plane at one design point on that surface.

In summarizing the findings of the numerical-search studies, two major points should be noted. First, the requirement for a two-phase redesign operation, coupled with the problem of step-size determination, leads to the conclusion that these procedures are computationally inefficient and not readily amenable to complete automation. Second, the biased-tangent and feasible-direction methods yield the same design as that achieved by the energy-based resizing method that aims for uniform flutter-velocity derivatives, thus giving added confidence in the superiority of this optimality criterion.

Selected Optimization Procedure

From the results presented in the previous two sections, it was concluded that the finally selected flutter resizing algorithm should be a direct rather than two-phase procedure that achieves a state of uniform flutter-velocity derivatives for flutter-critical elements. Moreover, for the overall problem of determining a near-minimum-weight design that satisfies both strength requirements and a minimum flutter-speed constraint (for one critical flutter mechanism), the resizing procedure should aim toward an optimality criterion with the following characteristics:

1) Flutter-critical elements have uniform flutter-velocity derivatives for the Mach number and altitude of the prescribed critical flutter condition; 2) strength-critical elements are fully stressed for at least one of the specified design loading conditions; and 3) other elements are at the minimum manufacturing gage.

The procedure described in the following subsection enforces this criterion and is applicable to both mass-balance and structural design variables.

Development and Evaluation of the Final Flutter-Resizing Equation

In the course of evaluating the energy-based methods, it was observed that the majority of structural variables resized for flutter were those for which the generalized strain-energy-density contribution in the expression for the flutter-velocity derivatives, Eq. (22a), was dominant. This observation suggested the possibility of revising Eq. (24) so as to rely exclusively on the total derivative expression of Eq. (22a):

$$m_{i_{\text{new}}} = m_{i_{\text{old}}} \left(\frac{(\partial V_f / \partial m_i)_{\text{old}}}{(\partial V_f / \partial m_i)_{\text{target}}} \right)^{1/2} \tag{33}$$

It was hoped that this resizing formula, hereafter referred to as the "velocity-derivative-ratio method," would continue to exhibit the good convergence characteristics of its predecessor for structural variables, while also being mathematically capable of dealing with mass-balance design variables.

By examining the resizing formula from the viewpoint of numerical-search methods, it can be shown that this equation implies that all velocity-constraint surfaces are hyperbolic in weight space and have the general character illustrated in Figs. 6 and 7. This approximation appeared intuitively reasonable and offered a potential improvement over the linearized interpretation implicit in the previous numerical-search procedures.

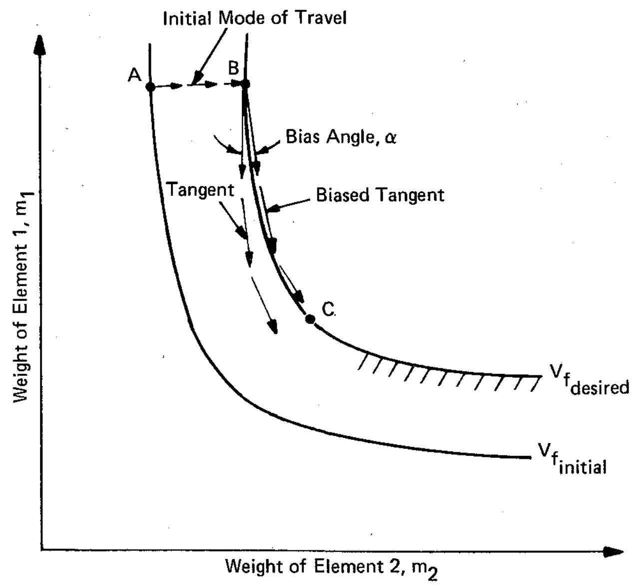

Fig. 7 Biased tangent vs tangent mode of travel.

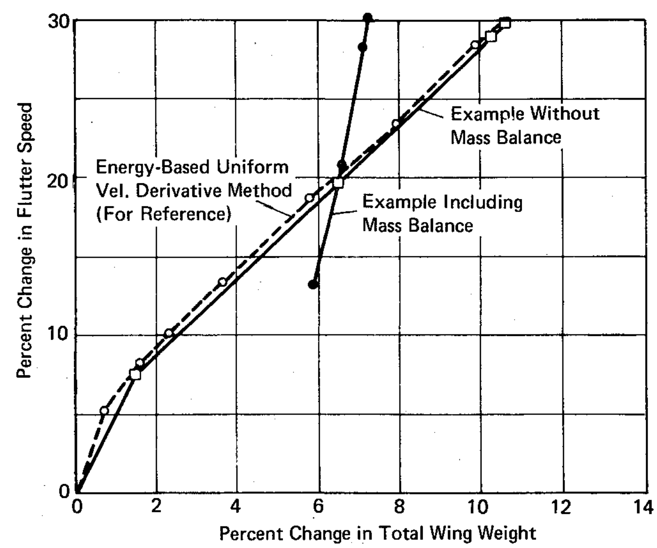

Fig. 8 Results of velocity-derivative-ratio method.

In applying Eq. (33) to the sample wing structure, two separate designs were investigated. One without mass-balance variables (i.e., the same problem treated previously); and one which includes a concentrated mass at the leading edge, (see Fig. 1). As before, interaction with strength and minimum-gage constraints was accounted for by simply not allowing the size of any element to fall below its initial value.

For the first problem, the design steps indicated in Fig. 8 show excellent convergence to the same region of optimum designs obtained by the energy-based method that seeks uniform flutter-velocity derivatives and by the numerical-search procedures. Figure 9 shows how the total weight increment of 10.7% of the initial structural weight is distributed among the flutter-critical elements in the final design. Also shown are the adjusted gages of these elements. The level of uniformity of the final flutter-velocity derivatives of the critical members may be observed in Fig. 10, which also shows the original values of these derivatives for the fully stressed design.

In applying this resizing approach, an iterative procedure was developed for determining the value of the target derivative used in Eq. (33). With a specified desired velocity increment, δV_f, application of the procedure starts with an assumed value for the target derivative and determines, according to Eq. (33) and subject to strength and minimum-gage

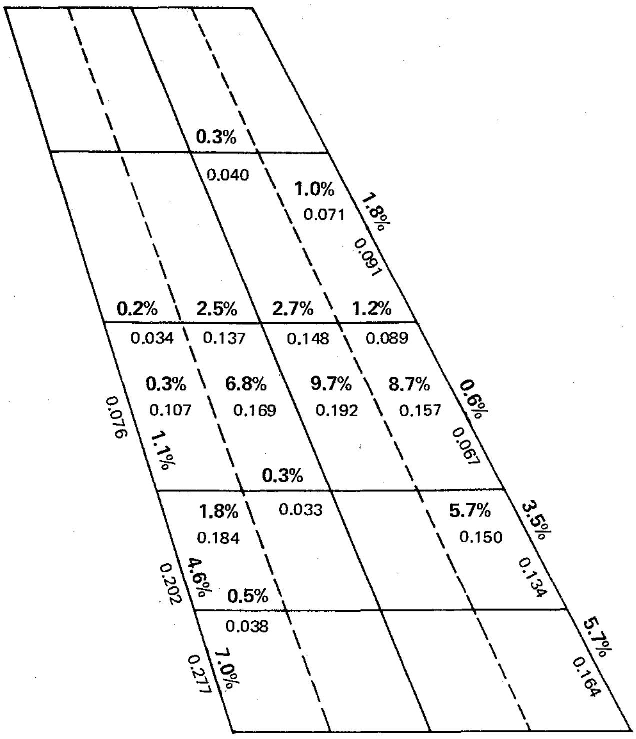

Fig. 9 Percent of total weight increment added to each element and final gages after redesign by the velocity-derivative-ratio method. Note: percentages shown for cover-skin elements are for one cover only.

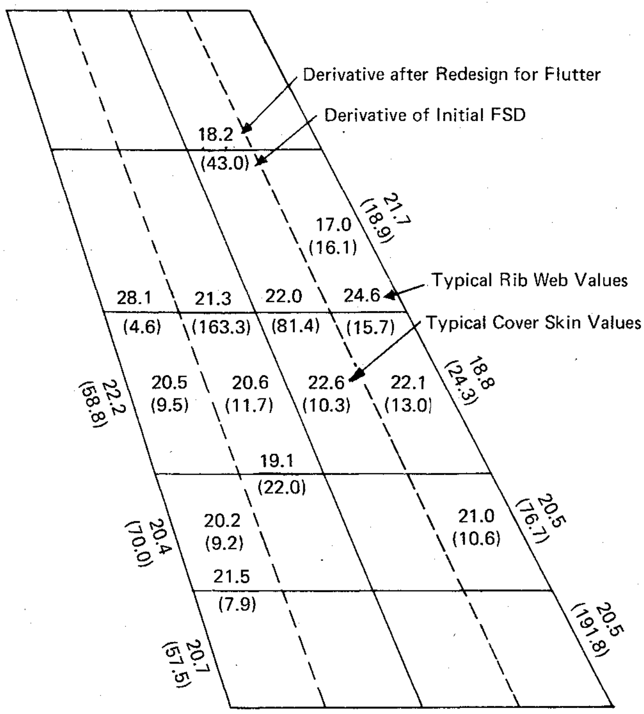

Fig. 10 Comparison of flutter-velocity derivatives of flutter-critical elements before and after redesign by the velocity-derivative-ratio method (kt/lb).

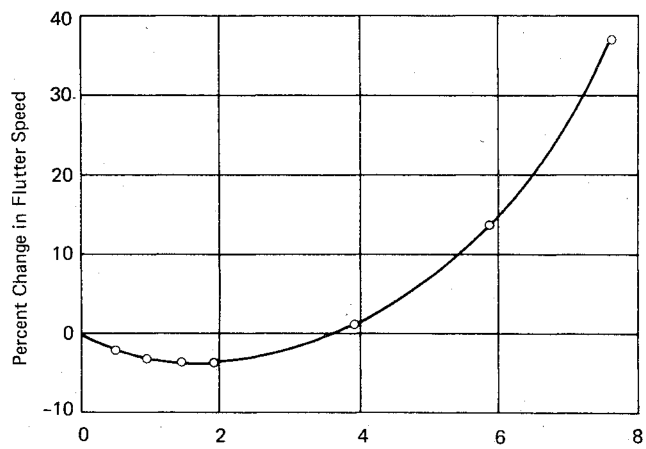

Fig. 11 Variation of flutter speed with mass balance.

constraints, what increments of weight will be assigned to each variable. Then, using the linear relationship of Eq. (26), a predicted increment of velocity is computed and compared with the desired value. If these are not within a specified tolerance, the target derivative is adjusted and the process is repeated until convergence is achieved. At this point, the element size changes are accepted and a new flutter velocity and a new set of derivatives are computed.

As a matter of practical interest, some comments are in order regarding the use of ratios of flutter-velocity derivatives in Eq. (33). Many element derivatives will be very small, or even negative, and, in such cases, it would appear desirable to reduce the element's size to its value dictated by strength or minimum-gage requirements. However, it was found that the stability of the resizing procedure was improved if the reduction in any element's size, in a redesign step, was limited to 25% of its previous value.

In the second redesign problem which included mass balance, it was found that the redesign behavior depended on the choice of initial value for the mass variable. For small values, the initial flutter-velocity derivative was negative and the procedure entirely eliminated mass balance from the final design. However, for large values of mass, the initial velocity derivative was positive and large, and mass balance became an active variable in redesign. This variation in flutter-velocity derivative is evident in Fig. 11 which shows change in flutter speed as a function of initial mass balance alone. In the final redesign case studied, a starting value of mass-balance weight equal to about 6% of the initial structural weight was selected resulting in rapid convergence to the desired flutter speed, as shown in Fig. 8, with a total weight increase of only about 7%. The final design contained only four resized structural elements consisting of forward and aft beam webs with the mass balance accounting for most of the total weight increase. Excellent uniformity of derivatives was achieved.

In summary, implementation of the velocity-derivative-ratio method was simple and direct in achieving minimum-weight designs for the cases studied, and no severe problems are anticipated in its application to large-scale structural idealizations. It is apparent, however, that it may sometimes be necessary to perform exploratory investigations to take advantage of the potential effectiveness of mass balance in the flutter redesign process. This may be accomplished by initially adding mass at different locations to determine whether the velocity derivatives can be increased to effective values.

Integration of the Combined Flutter and Strength Resizing Method

The preceding method for resizing a structure to meet flutter requirements has been incorporated into a comprehensive structural analysis and design system called FASTOP (Flutter And Strength Optimization Program). Interaction with strength requirements is achieved by successively optimizing for strength and flutter with the strength-designed members from the fully-stressed-design procedure (ASOP) being considered as minimum gages in the next flutter optimization, and vice versa.

Conclusion

The selected optimality criterion and its associated resizing algorithm have been demonstrated to offer a sound base upon which to build an automated procedure for dealing with the large-scale strength and flutter optimization problem. The simplicity of using flutter-velocity-derivative ratios to resize elements critical for flutter is consistent with the simplicity and practicality associated with the use of stress ratios in resizing for strength.

Appendix: Strain Energy During Oscillation in the Flutter Mode

The procedure used to arrive at the expression for the strain energy given in Eq. (4) is presented in this Appendix. For one unit of generalized flutter-mode displacement, the physical displacements, $\{x\}$, of the structural coordinates will vary with time according to

$$\{x\} = Re(\{U\}e^{i\omega_f t}) \tag{A1}$$

where $\{U\}$ is the arbitrarily normalized complex flutter mode shape and ω_f is the flutter frequency. Upon substituting $\{U_R\}+i\{U_I\}$ for $\{U\}$, and the equivalent trigonometric form of the exponential, Eq. (A1) becomes

$$\{x\} = \{U_R\}\cos\omega_f t - \{U_I\}\sin\omega_f t \tag{A2}$$

The total structural strain energy, E_s, is given in terms of these displacements and the structure's stiffness matrix, $[K]$, by

$$E_s = \tfrac{1}{2}\{x\}^T[K]\{x\} \tag{A3}$$

Substituting Eq. (A2) in (A3), noting the symmetry of $[K]$, and using the definitions given in Eqs. (5a-c), yields

$$\begin{aligned} E_s &= \tfrac{1}{2}[a\cos^2\omega_f t + b\sin^2\omega_f t + c\sin\omega_f t\cos\omega_f t] \\ &= \tfrac{1}{2}\left[a-(a-b)\left(\frac{1-\cos 2\omega_f t}{2}\right)+c\left(\frac{\sin 2\omega_f t}{2}\right)\right] \\ &= \tfrac{1}{2}\left[\left(\frac{a+b}{2}\right)+\left(\frac{a-b}{2}\right)\cos 2\omega_f t+\frac{c}{2}\sin 2\omega_f t\right] \end{aligned} \tag{A4}$$

which can be rewritten in the form of Eq. (4).

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